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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

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**INPUT INEFFICIENCY IN COMMERCIAL BANKS:
A NORMALIZED QUADRATIC INPUT DISTANCE APPROACH**

Thomas L. Marsh
Associate Professor
Department of Agricultural Economics
Kansas State University
Manhattan, KS, 66506
phone: 785-532-4913
fax: 785-532-6925
email: tlmarsh@agecon.ksu.edu

Allen M. Featherstone
Professor
Department of Agricultural Economics
Kansas State University
Manhattan, KS, 66506
email: afeather@agecon.ksu.edu

Thomas A. Garrett
Research Division
Federal Reserve Bank of St. Louis
St. Louis, Missouri 63102
email: tom.a.garrett@stls.frb.org

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Abstract: A normalized quadratic input distance function is proposed with which to estimate technical efficiency on commercial banks regulated by the Federal Reserve System. The study period covers 1990 to 2000 using individual bank information from the Call and Banking Holding Company Database. A stochastic frontier model is specified to estimate the input normalized distance function and obtain measures of technical efficiency.

INPUT INEFFICIENCY IN COMMERCIAL BANKS: A NORMALIZED QUADRATIC INPUT DISTANCE APPROACH

Introduction

In this paper we explore technical efficiency of commercial banks using an input distance function approach. The input distance function approach is of interest because it is a valid representation of multiple output technologies and directly measures technical efficiency in producing a given set of outputs. The analysis covers the sample period from 1990 to 2000 using Call and Banking Holding Company Database information for individual commercial banks. In the analysis, we implement a normalized quadratic distance function that characterizes multiple input and output production processes estimated with Bayesian econometrics. The Bayesian method provides a systematic approach for more efficient estimation by imposing parameter and economic restrictions, which are inherent in duality models of firm behavior.

Kaparakis, Miller, and Noulas (1994) provided a review of methodologies and conclusions for eight studies on bank frontier analysis. Past studies have taken non-parametric and parametric estimation approaches, including mathematical programming, stochastic frontier analysis, and simultaneous equation estimation. In addition, studies have used various functional forms such as the translog cost function (Ferrier and Lovell, 1990), profit function (Berger, et al. 1993), and output distance function (English, et al., 1993). The consensus of these studies is that significant inefficiencies exist and were generally declining over time (possibly due to deregulation), banks exhibit better allocative relative to technical inefficiency, and that external factors explains some of the observed inefficiencies. More recently, Berger and Mester (1999) found that cost productivity decreased while profit productivity increased from 1991-1997, particularly for banks involved in mergers. Wheelock and Wilson (2001) examined measures of scale and product mix economies with nonparametric

estimation found that banks experience increasing returns to scale up to approximately \$500 million dollars in assets. Reported efficiencies in past studies vary over a wide range and comparisons are difficult due to differences in maintained hypotheses, sample, and functional form.

Our methodological focus is on the production side where we specify a form of the normalized quadratic function exhibiting properties consistent with an input distance function. No study to date has explored technical efficiency in banking using input distance function approach. Furthermore, research on normalized quadratic distance functions is limited. On the consumer demand side, Holt and Bishop (2002) recently specified a normalized quadratic distance function and used it to estimate inverse demand relationships for fish. Also, the normalized quadratic input distance function is specified to accommodate both single and multiple output production processes and allows direct testing or imposition of input and output curvature conditions. Even for the case of a single input where the properties of the consumer and input distance function are equivalent (Cornes 1992), the functional specification is different.

To estimate measures of technical efficiency, we exploit the stochastic frontier approach (Stevenson 1980; Greene 1980, 1990; Battese and Coelli 1988). This framework coupled with the normalized quadratic function is sufficiently flexible to impose economic restrictions on both inputs and outputs with Bayesian estimation. We implement a parametric estimator that uses a maximum likelihood function to construct a Bayesian Markov chain Monte Carlo model with economic restrictions imposed following Geweke (1986). This research compliments recent studies by Atkinson and Primont (2002) and Atkinson, Färe, and Primont (2003), who estimated complete systems of inverse demand relationships jointly with the distance function using a GMM estimator.

The paper proceeds in the following manner. First, we briefly review the concepts underlying the input distance function and technical efficiency. Next, the normalized quadratic input distance

function is specified and extended to be consistent with the stochastic frontier framework. This is followed by specification of a Bayesian estimator for the exponential stochastic frontier model. Then, data and additional empirical methodology are discussed. Finally, results and discussion are provided with concluding comments.

Input Distance Function and Technical Efficiency

Input Distance Function

The direct input distance function is defined by

$$(1) \quad D(\mathbf{x}, \mathbf{y}) = \sup_{\delta} \{ \delta > 0 \mid (\mathbf{x} / \delta) \in S(\mathbf{y}), \forall \mathbf{y} \in \mathbf{R}_+^M \}$$

where $\delta \geq 1$. In (1), \mathbf{y} is a $(m \times 1)$ vector of outputs, $\mathbf{x} = (x_1, \dots, x_k)'$ is a $(n \times 1)$ vector of inputs and $S(\mathbf{y})$ is the set of all input vectors $\mathbf{x} \in \mathbf{R}_+^n$ that can produce the output vector $\mathbf{y} \in \mathbf{R}_+^m$. The underlying behavioral assumption is that the distance function represents a rescaling of all the input levels consistent with a target output level. Intuitively, δ is the maximum value by which one could divide \mathbf{x} and still produce \mathbf{y} . The value δ places \mathbf{x} / δ on the boundary of $S(\mathbf{y})$ and on the ray through \mathbf{x} . For example, in Figure 1, the distance function value is $D(x, y) = OB/OA$; the value required to scale the vector x^1 back to x^* on the boundary of $S(\mathbf{y})$. In other words, the input distance function measures the extent to which the firm is input efficient in producing a fixed set of output. Investigating the distance function is interesting because it is a dual representation of the cost function and both are valid representations of multiple output technologies.

The standard properties of a distance function are that it is homogenous of degree one, nondecreasing, and concave in input quantities \mathbf{x} , as well as nonincreasing and quasi-concave in outputs \mathbf{y} (Shephard 1970; Färe and Primont 1995). From (1) inverse factor demand equations may be obtained by applying Gorman's Lemma

$$(2) \quad \frac{\partial D(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}} = \mathbf{p}^*(\mathbf{x}, \mathbf{y})$$

where $\mathbf{p}^* = (p_1, \dots, p_n)'$ is a $(n \times 1)$ vector of cost normalized input prices or $p_i^* = p_i / \sum_{j=1}^n p_j x_j$. The

Hessian matrix is given by the second order derivatives of the distance function (Antonelli matrix)

$$(3) \quad A = \begin{bmatrix} \frac{\partial^2 D(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x} \partial \mathbf{x}'} & \frac{\partial^2 D(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x} \partial \mathbf{y}'} \\ \frac{\partial^2 D(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y} \partial \mathbf{x}'} & \frac{\partial^2 D(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y} \partial \mathbf{y}'} \end{bmatrix}$$

Imposing monotonicity constraints require that $\partial D(\mathbf{x}, \mathbf{y}) / \partial \mathbf{x} \geq \mathbf{0}$ and $\partial D(\mathbf{x}, \mathbf{y}) / \partial \mathbf{y} \leq \mathbf{0}$, while curvature constraints are based on the eigenvalues of the Antonelli matrix in (3).

Technical Efficiency

The input distance function has been exploited as a measure of technical efficiency (Farrell 1957; Debreu 1951). Inefficiencies arise if firms do not use cost minimizing amounts of input for several reasons, including regulated production, production quotas, or shortages (Atkinson and Primont 2002; Atkinson, Färe, and Primont 2003). The input-oriented measures of technical efficiency are given by

$$(4) \quad TE = 1/D = \inf_{\delta} \{ \delta : \delta \mathbf{x} \in S(\mathbf{y}) \}$$

where TE lies between zero and one. This efficiency measure can be equivalently specified as

$$(5) \quad \ln D + \ln TE = \ln D - u = 0$$

where the term $u = -\ln TE$ can be expressed as $TE = \exp(-u)$. Hence, u is nonnegative being bounded below by zero and unbounded from above.

Normalized Quadratic Distance Function

To complete the empirical model specification, we specify a normalized quadratic distance function. The normalized quadratic allows explicit investigation of the interactions between inputs and outputs and allows imposition of curvature conditions. The importance of curvature properties was

emphasized by Berger, Hancock, and Humphery (1993). Featherstone and Moss (1994) used a normalized quadratic cost function with curvature properties to measure economies of scale and scope in agricultural banking, finding contrasting results in measures of scope and scale with or without curvature restrictions. The proposed normalized quadratic distance function is given by

$$(6) \quad D(\mathbf{x}, \mathbf{y}) = b_0 + \sum_{i=1}^n b_i x_i + \sum_{i=n+1}^{n+m} b_i y_i + \frac{1}{2} \left(\left(\sum_{k=1}^n \alpha_k x_k \right)^{-1} \sum_{i=1}^n \sum_{j=1}^n b_{ij} x_i x_j + \sum_{i=n+1}^{n+m} \sum_{j=n+1}^{n+m} b_{ij} y_i y_j \right) + \sum_{i=1}^n \sum_{j=n+1}^{n+m} b_{ij} x_i y_j$$

with n inputs and m outputs. The b_i 's and b_{ij} 's are parameters to be estimated, while the α_i are predetermined positive constants that dictate the form of normalization. Symmetry is imposed by restricting $b_{ij} = b_{ji}$. The normalized quadratic distance function in (6) is semiflexible at a reference vector \mathbf{x}^* (Diewert and Wales 1988).

Homogeneity of degree zero in inputs in the input demand equations implies that $\sum_{j=1}^n b_{ij} = 0$,

while the normalization restriction requires that $\sum_{k=1}^n \alpha_k x_k = 1$ at a reference vector. Normalizing

quantities by their mean values yields unit means, or $\mathbf{x}^* = (1, \dots, 1)' = \mathbf{l}_n$, which can be used as a reference

bundle. At a reference vector \mathbf{x}^* , the demand restrictions become

$$(7) \quad \sum_{k=1}^n \alpha_k x_k^* = \sum_{k=1}^n \alpha_k = 1, \alpha_k \geq 0, \forall k, \text{ and } \sum_{j=1}^n x_j^* b_{ij} = \sum_{j=1}^n b_{ij} = 0$$

Stochastic Input-Normalized Distance System

Given the distance function is homogeneous of degree one quantities, then it is possible to normalize by some λ (e.g., an input or output or convex combinations),

$$(8) \quad \frac{1}{\lambda} D(\mathbf{x}, \mathbf{y}) = D^* \left(\frac{\mathbf{x}}{\lambda}, \mathbf{y} \right) \Leftrightarrow \ln D(\mathbf{x}, \mathbf{y}) - \ln \lambda = \ln D^* \left(\frac{\mathbf{x}}{\lambda}, \mathbf{y} \right)$$

From (5) the relationship can be rewritten as

$$(9) \quad \ln \lambda = -\ln D^* \left(\frac{\mathbf{x}}{\lambda}, \mathbf{y} \right) + u$$

In empirical applications, the term $u = -\ln TE$ has been exploited to form an estimable equation of the distance function itself that provides a direct measure of input inefficiency (Stevenson 1980; Greene 1980; Battese and Coelli 1988; Morrison Paul, Johnston, and Frengley 2000; Brümmer, Glauben, and Thussen 2002).

To define a distance function normalized by the k th input let $x_s^* = x_s / x_k \forall s = 1, \dots, n$. Define the predetermined constants as $\alpha = (0, \dots, 0, \alpha_k, 0, \dots, 0) \ni \alpha_k = 1$, then $\sum_{s=1}^n \alpha_s x_s^* = 1$. Using the homogeneity property of the distance function, it can be written as

$$(10) \quad D^*(\mathbf{x}, \mathbf{y}) = \frac{D(\mathbf{x} / x_k, \mathbf{y})}{x_k} = b_0^* + \sum_{i=1}^{n-1} b_i^* x_i^* + \sum_{i=n+1}^{n+m} b_i y_i + \frac{1}{2} \left(\sum_{i=1}^{n-1} \sum_{j=1}^{n-1} b_{ij}^* x_i^* x_j^* + \sum_{i=n+1}^{n+m} \sum_{j=n+1}^{n+m} b_{ij} y_i y_j \right) + \sum_{i=1}^{n-1} \sum_{j=n+1}^{n+m} b_{ij} x_i^* y_j$$

Hence, the distance function in (10) is a special case of that in (6). From (9) the k th input-normalized distance function can be represented by

$$(11) \quad \ln x_k = -\ln \left(b_0^* + \sum_{i=1}^{n-1} b_i^* x_i^* + \sum_{i=n+1}^{n+m} b_i y_i + \frac{1}{2} \left(\sum_{i=1}^{n-1} \sum_{j=1}^{n-1} b_{ij}^* x_i^* x_j^* + \sum_{i=n+1}^{n+m} \sum_{j=n+1}^{n+m} b_{ij} y_i y_j \right) + \sum_{i=1}^{n-1} \sum_{j=n+1}^{n+m} b_{ij} x_i^* y_j \right) + u + \varepsilon_0$$

where ε_0 is assumed to be an identically distributed stochastic error term and independent of u .

Estimation issues concerning (11) are complicated by that fact that u is unobserved, but have been addressed in several ways in the stochastic frontier production literature, which we discuss in more detail below.

Econometric Estimation

Following Greene (1980, 1990) the likelihood for the composite error term v is specified as a GAMMA distribution with parameters $\theta > 2$ and $\lambda = 1$, which yields the exponential

distribution. For $\theta > 2$ the maximum likelihood estimation of the parameters is a regular case.

The log-likelihood function for (11) becomes

$$(13) \quad \ln L(\beta, \theta, \sigma | \mathbf{Y}, \mathbf{X}) = \sum_{i=1}^T \left\{ \ln \theta + (\theta \sigma)^2 / 2 + \theta \varepsilon_i + \ln \Phi \left[(-\varepsilon_i - \theta \sigma^2) / \sigma \right] \right\}$$

where σ is the variance of the normal distribution. Under a general set of regularity conditions the maximum likelihood estimates are asymptotically normally distributed and asymptotically efficient.

Markov Chain Monte Carlo

To specify a posterior pdf for either (12) or (13), we assume prior information on the $(\beta', \varphi)'$ with prior pdf $\pi(\theta, \varphi) = \pi(\beta)\pi(\varphi)$. Here, φ represents parameters σ and $\theta > 2$ in (13). The β parameters are assumed to have a noninformative prior (i.e., $\beta \propto \text{constant}$) for either model. For the truncated normal distribution μ is assumed to have uniform distribution bounded below by zero. The inverted gamma is used for a prior on σ , while θ is assumed to have a uniform distribution bounded below by two. These priors have been used in numerous Bayesian studies (e.g., Zellner, Bauwens, and Van Dijk 1988). The posterior pdf is then as

$$p(\beta, \varphi) = L(\beta, \sigma, \mu, \sigma_u | \mathbf{Y}, \mathbf{X}) \pi(\beta) \pi(\varphi)$$

Techniques of Markov chain Monte Carlo (MCMC) simulation estimation using the Metropolis-Hastings algorithm are applied to Bayesian estimation (Mittelhammer, Judge, and Miller 2000; Chib and Greenberg).

Empirical Methodology and Data

To estimate a measure of technical inefficiency a theoretically consistent model must be specified. There are two common approaches to modeling banks, the production and intermediation approach. The production approach measures bank production in terms of the

numbers of loans and deposit accounts serviced and includes operating costs. The intermediation approach measures outputs in terms of the dollar amounts of loans and deposits and includes operation costs and interest expense. We choose to follow the intermediation approach as have Berger et al (1987), Ferrier and Lovell (1990), Kaparakis, Miller, and Noulas (1994), and Wheelock and Wilson (2001) among others.

The data are from the 1990, 1994 and 2000 Call Report information for commercial banks. Following Kaparakis, Miller, and Noulas (1994) and Wheelock and Wilson (2001) the model includes four outputs, four variable inputs, and one quasi-fixed input. Outputs include loans to individuals (y_1), real estate loans (y_2), commercial and industrial loans (y_3), and federal funds, securities purchased under agreements to resell (y_4). Inputs include interest-bearing deposits except certificates of deposits greater than \$100,000 (x_1), purchased funds (certificates of deposits greater than \$100,000, federal funds purchased, and securities sold plus demand notes) and other borrowed money (x_2), number of employees (x_3), and book value of premises and fixed assets (x_4). The quasi-fixed asset is noninterest-bearing bonds. Kaparakis, Miller, and Noulas (1994) suggest that banks cannot attract more noninterest-bearing deposits by offering interest and they should be regarded as exogenous. The data used in the empirical model are based on average quarterly values across a given year.

Rather than compute input prices, we choose to estimate only the distance function itself in (11) without the system of inverse demand relationships defined by (2). Typically, inverse demand relationships are included to increase econometric efficiency, obtain measures of price flexibilities, or obtain dual cost measures. Our justification is that for large sample sizes the efficiency gains from including the inverse demand system will likely not compensate for the added numerical complexities and computations, and because our interest is technical efficiency

that is completely characterized by (11). Moreover, including calculated input prices may introduce measurement error or results in prices with little price variation that can compromise empirical duality properties (Lusk, Featherstone, Marsh, and Abdulkadri).

To arrive at the final data sets for estimation, several data management steps were taken. First, we excluded banks that reported negative inputs or outputs (which only influenced x_1). This yielded 12,395 observations in 1990, 10,765 observations in 1994, and 8,517 observations in 2000. Then to account for extreme outliers, we excluded banks that 6 or more standard deviations away from the mean of the input and output values. In 1990 there were 12,218 remaining observations, in 1994 there were 10,620 remaining observations, and in 2000 there were 8,409 remaining observations. The number of employees (x_3) was used to normalize the other inputs because it had a few reported zero values (e.g., in 2000 there were only eleven zero values). The zero values were assigned the minimum value of the remaining observations in x_3 .

Econometric models of (11) were estimated for each year using the Bayesian estimator based on alternative cross-sections of the data. Models were estimated on the entire data set, for banks with total assets less than \$50 million, and banks with assets greater than \$50 million. Partitioning data in this manner are consistent with previous studies (e.g., Kaparakis, Miller, and Noulas 1994) and allows comparison and testing of results between smaller and larger banks (as well as across the entire sample). A histogram of the number of banks across total assets is presented in Figure 2, showing a steady decrease (increase) in the number of banks with total assets under (over) \$50 million.

To complete the MCMC simulation of the Bayesian estimator, a burn-in period of 30,000 iterations was used. These iterations were then discarded and 70,000 additional iterations were simulated to yield the final empirical distribution. Additional details of the data and the MCMC

analysis are available from the authors upon request. Curvature conditions are imposed using Cholesky decomposition (Lau 1970).

Results and Discussion

Empirical results are presented in Table 1 for 1990, 1994, and 2000. For convenience we summarize these results with the median, mean, and standard deviation of technical efficiency in Table 1 for the Bayesian exponential model.

In general, the preliminary technical efficiency estimates are consistent with those obtained in Berger et al. (1993) and English, et al. (1993). English et al. (1993) report a mean output technical efficiency of 0.754 with standard deviation of 0.145 for small commercial banks in 1982. Focusing on the results from the exponential model over the entire sample, input efficiency has increased over the sample period and were higher for larger banks. In 1990 and 1994, the median efficiency values were nearly identical yielding 0.732 and 0.730, respectively. In 2000, the median efficiency level over the entire sample increased to 0.754. For smaller banks (total assets less than \$50 million) the median technical efficiency measure incremented from 0.696 in 1990, to 0.704 in 1994, and to 0.715 in 2000. For larger banks (total assets greater than \$50 million) the median technical efficiency measure increased from 0.75 in 1990 and leveled off to 0.80 in 1994 and 2000. Comparing across bank sizes, larger banks were more 7%, 14%, and 11% more efficient than smaller banks in 1990, 1994, and 2000 respectively. Note that, when comparing the mean technical efficiency measures, the differences would reduce to 0%, 6%, and 4% in 1990, 1994, and 2000 respectively. In all, these results are consistent with the interpretation that bank efficiency has been increasing over time (Kaparakis, Miller, and Noulas 1994) and that the larger banks exhibit higher technical efficiency levels (Berger, et al. 1993).

Results were also obtained by estimating (11) without curvature restrictions in 2000, providing mixed results. For smaller banks, relaxing curvature conditions increased technical efficiency. For

larger banks, relaxing curvature conditions decreased technical efficiency. Across the entire sample, the technical efficiency measures were nearly identical. Although the results are mixed, it is apparent that technical efficiency results are sensitive to curvature restrictions. However, the direction of this effect was not consistent between smaller and larger banks.

Conclusions

In this paper a normalized quadratic input distance function is proposed with which to estimate technical efficiency on commercial banks regulated by the Federal Reserve System. The study period covers 1990 to 2000 using individual bank information from the Call and Banking Holding Company Database. A Bayesian variation of a stochastic frontier model is used to estimate the input normalized distance function and obtain measures of technical efficiency. Preliminary findings based on 1990, 1994, and 2000 data are consistent with previous findings in that technical inefficiency appears to be decreasing over time and that larger banks are more efficient. We recognize limitations of the research presented in this paper. Perhaps most importantly, technical efficiency estimates were based only on selected years. Our intention is to revisit and extend the empirical analysis by using a panel data set from 1990 to 2000. In addition, previous research suggests that important determinants of technical efficiency may be exogenous to the variables specified in the input distance function. Following Atkinson and Primont (2002) and Atkinson, Färe, and Primont (2003), the intention is to investigate a fixed effect approach where technical efficiency is model as a function of these exogenous factors.

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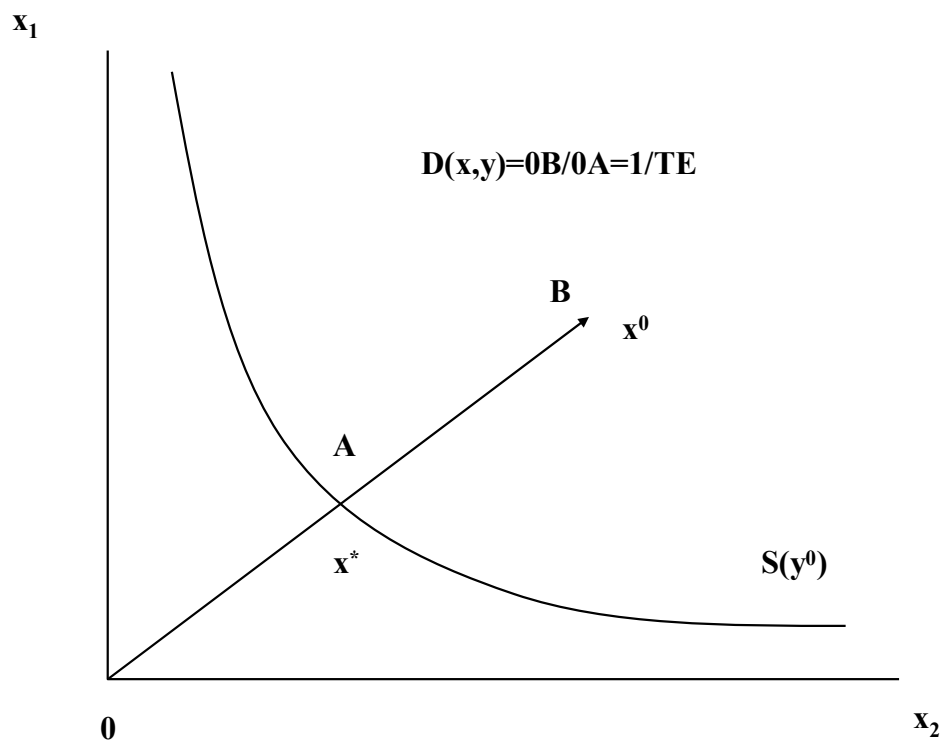


Figure 1. The Distance Function and Input Efficiency.

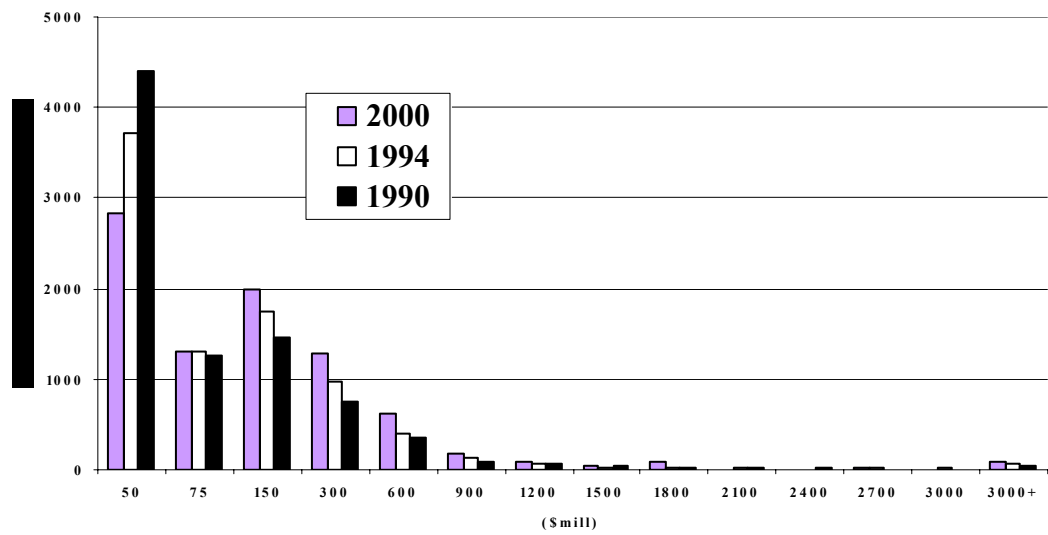


Figure 2. Number of Commercial Banks by Total Assets.

Table 1. Technical Efficiency Measures from Stochastic Exponential Model

Small Commercial Banks				
Year	Median	Mean	Standard Deviation	
1990	0.696	0.696	0.113	
1994	0.704	0.709	0.112	
2000	0.715	0.716	0.100	w/curvature
	0.738	0.739	0.096	wo/curvature
Large Commercial Banks				
	Median	Mean	Standard Deviation	
1990	0.754	0.705	0.141	
1994	0.800	0.750	0.154	
2000	0.801	0.749	0.166	w/curvature
	0.768	0.721	0.164	wo/curvature
All Commercial Banks				
	Median	Mean	Standard Deviation	
1990	0.732	0.683	0.155	
1994	0.730	0.685	0.145	
2000	0.754	0.703	0.161	w/curvature
	0.755	0.704	0.162	wo/curvature